Optimizing Recursive Queries with Monotonic Aggregates in $DeALS$

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***Abstract*—The exploding demand for analytics has refocused the attention of data scientists on applications requiring aggregation in recursion. After resisting the efforts of researchers for more than twenty years, this problem is being addressed by innovative systems that are raising logic-oriented data languages to the levels of generality and performance that are needed to support efficiently a broad range of applications. Foremost among these new systems, the *Deductive Application Language System* ($DeALS$) achieves superior generality and performance via new constructs and optimization techniques for monotonic aggregates which are described in the paper. The use of a special class of monotonic aggregates in recursion was made possible by recent theoretical results that proved that they preserve the rigorous least-fixpoint semantics of core Datalog programs. This paper thus describes how $DeALS$ extends their definitions and modifies their syntax to enable a concise expression of applications that, without them, could not be expressed in performance-conducive ways, or could not be expressed at all. Then the paper turns to the performance issue, and introduces novel implementation and optimization techniques that outperform traditional approaches, including *Semi-naive* evaluation. An extensive experimental evaluation was executed comparing $DeALS$ with other systems on large datasets. The results suggest that, unlike other systems, $DeALS$ indeed combines superior generality with superior performance.**

I. Introduction

The fast-growing demand for analytics has placed renewed focus on improving the support for aggregation in recursion. Aggregates in recursive programs are essential in many important applications, including programs in new areas such as computer networking [1], social networks [2], and data mining. Significant applications include machine learning-algorithms for Markov chains and hidden Markov models, and algorithms such as Apriori that require iterating over counts, or probability-computations. Besides these new applications, we can mention a long list of traditional ones, such as Bill of Materials (BOM), a.k.a. subparts explosion: this classical recursive query for DBMS requires aggregating the various parts in the part-subpart DAG. Finally, we have problems, such as computing the shortest paths or counting the number of paths between vertices in a graph, which are now covered as foundations by most CS101 textbooks.

Although aggregates were not covered in E.F. Codd's definition of the relational calculi [3], it did not take long before early versions of relational languages such as SQL included support for aggregate functions, namely count, sum, avg, min and max, along with associated constructs such as group-by. However, a general extension of recursive query theory and implementation to allow for aggregates proved an elusive goal, and even recent versions of SQL that provide strong support for OLAP and other advanced aggregates disallow the use of aggregates in recursion and only support queries that are stratified w.r.t. to aggregates.

Yet the desirability of extending aggregates to recursive queries was widely recognized early and many partial solutions were proposed over the years for Datalog languages [4], [5], [6], [7], [8], [9], [10]. The fact that, in general, aggregates are non-monotonic w.r.t. set-containment led to proposals based on non-monotonic theories, such as locally stratified programs and perfect models [11], [12], well-founded models [13] and stable models [14]. An alternative approach was due to Ross and Sagiv [9], who observed that particular aggregates, such as continuous count, are monotonic over lattices other than set-containment and thus can be used in non-stratified programs. However practical difficulties with this approach were soon pointed out by [15] who showed that determining the correct lattices for programmers and compilers would be quite difficult, and that prevented the deployment of the monotonicity idea in practical query languages for a long time. Fortunately, we recently witnessed some dramatic developments change the situation completely. The first is that Hellerstein et al., after announcing a resurgence of Datalog, showed that monotonicity in special lattices can be very useful in proving the formal properties such as eventual consistency [16]. Then, we have seen monotonic aggregates making a strong come back in practical query languages thanks to the results published in [17], [18] and in [2], summarized below.

The formalization of monotonic aggregates proposed in [17], [18] preserves monotonicity w.r.t. set containment, and it is thus conducive to simplicity and performance that follow respectively from the facts that (i) users no longer have to deal with lattices, and (ii) the query optimization techniques, such as *Semi-naive* and magic sets remain applicable [17]. SociaLite [2] also made an important contribution by showing that shortest-path queries, and other algorithms using aggregates in recursion, can be implemented very efficiently so that in many situations the Datalog approach becomes preferable to that of hand-coding big-data analytics in some procedural language.

These dramatic advances represented a major source of

opportunities and challenges for our ***De**ductive **A**pplication **L**anguage* ($DeAL$) and its system $DeALS$. In fact, unlike the design of the SociaLite system where the performance of recursive graph algorithms with aggregates had played a role, $DeALS$ has been designed as a very general system seeking to satisfy the many needs and lessons that had been learned in the course of a long experience with logic-based data languages, and the LDL [19] and LDL++ [20] experiences in particular. Thus, $DeALS$ supports key non-monotonic constructs having formal stable model semantics, including, e.g., XY-stratification and the choice construct that were found quite useful program analysis [21], and user-defined aggregates that enabled important knowledge-discovery applications [22].

In addition to a rich set of constructs, $DeALS$ was also designed to support a roster of optimization techniques including magic sets and supplementary magic sets, non-linear recursion, and existential quantification. Introducing powerful new constructs and their optimization techniques by retrofitting a system that already supports a rich set of constructs and optimizations represented a difficult technical challenge. In this paper, we describe how this challenge was met with the introduction of new optimization techniques for monotonic aggregates. We will show that $DeALS$ now achieves both performance and generality, and we will underscore this point by comparing not only with SociaLite but also with systems such as DLV [23] and LogicBlox [24] that realize different performance/generality tradeoffs.

Overview Preliminaries are reviewed in the next section. We then begin the first of two main parts of the paper. Section III presents the syntax and semantics for our min (`mmin`) and max (`mmax`) monotonic aggregates. Section IV discusses the evaluation and optimization of monotonic aggregate programs. Section V presents implementation details for `mmin` and `mmax` and our B+Tree based storage manager followed by its experimental validation in Section VI. The second part of the paper begins with Section VII discussing the novel optimization techniques developed for the count (`mcount`) and sum (`msum`) monotonic aggregates, followed by their implementation in Section VIII and experimental validation in Section IX. Section X provides additional $DeAL$ program examples. Section XI presents the formal semantics on which our aggregates are based. Related work is reviewed in Section XII and we conclude in Section XIII.

II. Preliminaries

Datalog Programs A Datalog program P is a finite set of rules, or Horn Clauses, where rule r in P has the form $A \leftarrow A_1, \ldots, A_n$. The *atom* A is the $head$ of r. $A_1, \ldots, A_n$, the $body$ of r, are *literals*, or *goals*, where each literal can be either a positive or negated atom. An atom has the form $p(t_1, \ldots, t_j)$ where p is a *predicate* and $t_1, \ldots, t_j$ are *terms* which can be *constants*, *variables* or *functions*. An r with an empty body is a $fact$. A successful assignment of all variables of rule body goals results in a successful derivation for the rule head predicate. Datalog programs use set semantics and are (typically) *stratified* (i.e. partitioned into levels based on rule dependencies) and executed in level-by-level order, in a bottom-up fashion. Datalog programs can be evaluated using an iterative approach such as *Semi-naive evaluation* [10].

III. MMIN and MMAX Monotonic Aggregates

An `mmin` or `mmax` monotonic aggregate rule has the form:

$$\mathtt{p}(\mathtt{K_1}, \ldots, \mathtt{K_m}, \mathtt{aggr}\langle \mathtt{T} \rangle) \leftarrow \mathtt{Rule\ Body}.$$

$\mathtt{K_1}, \ldots, \mathtt{K_m}$ are the zero or more *group-by arguments* we also refer to as $\overline{\mathtt{K}}$. $\mathtt{aggr} \in \{\mathtt{mmax}, \mathtt{mmin}\}$ is the *monotonic aggregate*. `T` is the *aggregate term* which is a variable.

`mmin` and `mmax` are aggregate functions that map an input set or multiset, we will call G, to an output set, we will call D. Then, given G, for each element $g \in G$ `mmin` will put g into output set D if g is *less than the least value* `mmin` has previously computed for G. Similarly, given an input set G, for each element $g \in G$ `mmax` will put g in output set D if g is *greater than the greatest value* `mmax` has previously computed for G. When viewed as a sequence, the values produced by `mmin` and `mmax` is monotonic. The `mmin` and `mmax` aggregates are monotonic w.r.t. set-containment and can be used in recursive rules, therefore G can be viewed as a union of rule bodies *across* fixpoint iterations. These aggregates memorize the most recently computed value and thus require a single pass[1] over G.

A. Running Example

The All-Pairs Shortest Paths (APSP) program has received much attention in the literature [6], [9], [13], [25], [26]. APSP calculates the shortest distance path between each pair of connected vertices in a directed graph with edge costs.

Example 1: APSP with `mmin`

r1. $\mathtt{spaths}(\mathtt{X}, \mathtt{Y}, \mathtt{mmin}\langle \mathtt{D} \rangle) \leftarrow \mathtt{edge}(\mathtt{X}, \mathtt{Y}, \mathtt{D})$.
r2. $\mathtt{spaths}(\mathtt{X}, \mathtt{Y}, \mathtt{mmin}\langle \mathtt{D} \rangle) \leftarrow \mathtt{spaths}(\mathtt{X}, \mathtt{Z}, \mathtt{D1}), \mathtt{edge}(\mathtt{Z}, \mathtt{Y}, \mathtt{D2}), \mathtt{D} = \mathtt{D1} + \mathtt{D2}$.
r3. $\mathtt{shortestpaths}(\mathtt{X}, \mathtt{Y}, \mathtt{min}\langle \mathtt{D} \rangle) \leftarrow \mathtt{spaths}(\mathtt{X}, \mathtt{Y}, \mathtt{D})$.

Example 1 is the $DeAL$ APSP program with the `mmin` aggregate. The `edge` predicate denotes the edges of the graph. The intuition for this program is as follows. In the recursion (*r1*, *r2*), an `spaths` fact will be derived if a path from `X` to `Y` is either i) new or ii) has length shorter than the currently known length from `X` to `Y`. *r1* finds the shortest path for each `edge`. *r2* is the left-linear recursive rule that computes new shortest paths for `spaths` by extending previously derived paths in `spaths` with an `edge`. Logically, this approach can result in many facts `spaths` for `X`, `Y`, each with a different length. Therefore, the program is stratified using a traditional (non-monotonic) `min` aggregate (*r3*) to select the shortest path for each `X`, `Y`.

APSP By Example Next, we walk through a *Semi-naive* evaluation of the Shortest Paths program from Example 1.

First, *r1* in Example 1, the exit rule, is evaluated on the edge facts in Figure 1. In the rule head in *r1*, `X` and `Y`, the

[1]SQL 2003 max, min, count and sum aggregates on the `unlimited preeceding` window are similar to $DeAL$'s monotonic aggregates.

$\texttt{edge}(\texttt{a},\texttt{b},1).\ \ \texttt{edge}(\texttt{a},\texttt{c},3).\ \ \texttt{edge}(\texttt{a},\texttt{d},4).$
$\texttt{edge}(\texttt{b},\texttt{c},1).\ \ \texttt{edge}(\texttt{b},\texttt{d},4).\ \ \texttt{edge}(\texttt{c},\texttt{d},1).$

Fig. 1. edge Facts for Example 1

non-aggregate arguments, are the *group-by arguments*. The mmin aggregate is applied to each of the six edge facts and six spaths facts are successfully derived (not displayed for reasons of space) because no aggregate values had been previously computed (memorized) and each group (i.e. $(\texttt{a},\texttt{b})$) was represented among the facts only once. For the spaths predicate, mmin is now initialized with a value for each group.

$$\begin{aligned}\texttt{spaths}(\texttt{a},\texttt{c},2) &\leftarrow \texttt{spaths}(\texttt{a},\texttt{b},1),\texttt{edge}(\texttt{b},\texttt{c},1),2{=}1{+}1.\\ \texttt{FAIL} &\leftarrow \texttt{spaths}(\texttt{a},\texttt{b},1),\texttt{edge}(\texttt{b},\texttt{d},4),5{=}1{+}4.\ \ [\text{i}]\\ \texttt{FAIL} &\leftarrow \texttt{spaths}(\texttt{a},\texttt{c},3),\texttt{edge}(\texttt{c},\texttt{d},1),4{=}3{+}1.\ \ [\text{ii}]\\ \texttt{spaths}(\texttt{b},\texttt{d},2) &\leftarrow \texttt{spaths}(\texttt{b},\texttt{c},1),\texttt{edge}(\texttt{c},\texttt{d},1),2{=}1{+}1.\end{aligned}$$

Fig. 2. Derivations of Example 1 *r2* - Iteration 1

Semi-naive evaluates the recursive *r2* rule from Example 1 using the six spaths derived by *r1*. Figure 2 displays four derivations attempted by *r2* in its first iteration. Derivations not displayed failed to join edge and spaths facts. The first attempt results in a new spaths fact because $\texttt{spaths}(\texttt{a},\texttt{c},2)$ has an aggregate value less than the previous aggregate value for $(\texttt{a},\texttt{c})$, which was 3 (from *r1*). The failures denoted [i] and [ii] occurred because the facts to be derived would have aggregate values not less than the previous value for $(\texttt{a},\texttt{d})$, which is 4. Finally, $\texttt{spaths}(\texttt{b},\texttt{d},2)$ is derived ($2{<}4$ for $(\texttt{b},\texttt{d})$).

$$\texttt{spaths}(\texttt{a},\texttt{d},3) \leftarrow \texttt{spaths}(\texttt{a},\texttt{c},2),\texttt{edge}(\texttt{c},\texttt{d},1),3{=}2{+}1.$$

Fig. 3. Derivations of Example 1 *r2* - Iteration 2

Using the two facts derived in Figure 2, *Semi-naive* performs a second iteration using *r2*. As displayed in Figure 3, $\texttt{spaths}(\texttt{a},\texttt{d},3)$ is derived because $(3 < 4)$ for $(\texttt{a},\texttt{d})$. Now, no new facts can be derived and a fixpoint is reached.

$$\begin{aligned}&\texttt{shortestpaths}(\texttt{a},\texttt{c},2) \leftarrow \{\texttt{spaths}(\texttt{a},\texttt{c},3),\texttt{spaths}(\texttt{a},\texttt{c},2)\}\\ &\texttt{shortestpaths}(\texttt{a},\texttt{d},3) \leftarrow \{\texttt{spaths}(\texttt{a},\texttt{d},4),\texttt{spaths}(\texttt{a},\texttt{d},3)\}\\ &\texttt{shortestpaths}(\texttt{b},\texttt{d},2) \leftarrow \{\texttt{spaths}(\texttt{b},\texttt{d},4),\texttt{spaths}(\texttt{b},\texttt{d},2)\}\end{aligned}$$

Fig. 4. Derivations of Example 1 *r3*

Lastly, *r3* is evaluated over the spaths facts derived during recursion and uses a stratified min aggregate to derive only the fact with the shortest path for each group. Figure 4 displays derivations of *r3* on groups that had multiple facts derived in recursion showing why rules like *r3* are necessary with our semantics. In Section IV, we will discuss optimizations so rules such as *r3* do not have to be evaluated.

IV. Monotonic Aggregate Evaluation

In this section, we present optimized evaluation techniques for programs with monotonic aggregates. We start with a review of *Semi-naive* fixpoint evaluation, the technique that serves as the basis for our optimized evaluation approaches.

In Figure 5, the algorithm for *Semi-naive*, M is the initial model (database), S contains all facts obtained thus far, δS and $\delta S'$ contain facts obtained during the previous and current iteration, respectively, and T_E and T_R are the Immediate

1: $S := M$;
2: $\delta S := T_E(M)$;
3: $S := S \cup \delta S$;
4: **while** $\delta S \neq \varnothing$ **do**
5: $\quad \delta S' := T_R(\delta S) - S$;
6: $\quad S := S \cup \delta S'$;
7: $\quad \delta S := \delta S'$;
8: **return** S;

Fig. 5. Algorithm for *Semi-naive Evaluation*

Consequence Operator (ICO) for the exit rule(s) and the recursive rule(s), respectively. The algorithm evaluates as follows. Firstly, *Semi-naive* applies T_E (i.e. the exit rules) on M to derive the first set of new δ facts δS (line 2). Then, until no new facts are derived during an iteration, *Semi-naive* evaluates T_R on δS to derive new facts to be used in the next iteration. The new set of δ facts ($\delta S'$) is produced only after the removal of facts found in previous steps (line 5).

Symbolic differentiation rules [10] can be applied to monotonic aggregate rules in a straightforward manner to produce rules for *Semi-naive*. We omit this in the interest of space.

Although *Semi-naive* is an efficient evaluation technique for general Datalog programs, the *max-based optimization* [18] identified counting only needs to be performed on maximum (max) values if only monotonic arithmetic and boolean functions are used. In this work, we expand this observation which we refer to as the *Monotonic Optimization*. The intuition behind the *Monotonic Optimization* is that with our monotonic aggregates, monotonicity is preserved and values other than the max (mmax) or min (mmin) will add no new results and thus can be ignored. Only the max (min) intermediate values need to be used in derivations to produce the final max (min) value. In fact, the last fact produced by the aggregate for a group contains the greatest (mmax) or least (mmin) aggregate value, making this fact the only fact for the group that we need to produce for the next iteration.

A. Monotonic Aggregate Semi-naive Evaluation

1: $S := M$;
2: $\delta S := getLast(T_E(M))$;
3: $S := S \cup \delta S$;
4: **while** $\delta S \neq \varnothing$ **do**
5: $\quad \delta S' := getLast(T_R(\delta S)) - S$;
6: $\quad S := S \cup \delta S'$;
7: $\quad \delta S := \delta S'$;
8: **return** S;

Fig. 6. *Monotonic Aggregate Semi-naive Evaluation*

The *Monotonic Optimization* enables an optimized *Semi-naive* for monotonic aggregates we call *Monotonic Aggregate Semi-naive Evaluation* (*MASN*). Figure 6 is the algorithm for *MASN*, which closely resembles *Semi-naive*. *MASN*'s differences with *Semi-naive* are as follows. Here we use $getLast()$[2]

[2] $DeALS$ supports *MASN* by maintaining a single fact per group in $\delta S'$.

to produce, from the input set, a set containing i) all facts from predicates that do not have monotonic aggregates, and ii) the last derived fact for each group from monotonic aggregate predicates. Now after the T_E or T_R produces a set of facts, $getLast$ will be applied to produce the actual new $\delta S'$. Otherwise, *MASN* evaluation is the same as *Semi-naive*.

B. Eager Monotonic Aggregate Semi-naive Evaluation

MASN employs a level-by-level iteration boundary of a breadth-first search (BFS) algorithm. δ facts computed during the current iteration will be held for use until the next iteration. However, facts produced from monotonic aggregate rules can be used immediately upon derivation. Looking at the derivations in the walk-through evaluation of APSP in Section III-A one can see a case where *Semi-naive*, and in this case *MASN* as it would have evaluated the same as *Semi-naive*, did not capitalize on this property of monotonic aggregates.

Example 1 *r2* evaluation with *Semi-naive* or *MASN*
spaths(a, c, 2) $\leftarrow$ spaths(a, b, 1), edge(b, c, 1), 2 = 1+1.
FAIL $\leftarrow$ **spaths(a, c, 3)**, edge(c, d, 1), 4 = 3+1.

Fig. 7. Example of Iteration Boundary of *MASN*

Figure 7 shows the derivations of interest extracted from Figure 2. We see the second derivation performed using spaths(a, c, 3) (from δS) and result in failure as because the value for (a, d) was 4. However, at the time the derivation is attempted, spaths(a, c, 2), the result of the immediately previous derivation, existed. Had spaths(a, c, 2) been used, spaths(a, d, 3) would have been derived here, rather than requiring another *Semi-naive* iteration (Figure 3).

To capitalize on the *Monotonic Optimization*, we present *Eager Monotonic Aggregate Semi-naive Evaluation* (*EMSN*). With *EMSN*, facts produced from monotonic aggregate rules are available to be used immediately upon derivation. *EMSN* evaluates recursive rules with monotonic aggregates in a fact-oriented (fact-at-a-time) manner, but still uses a set to determine which groups are used in derivations each iteration. Derivations with monotonic aggregates are always performed with the current aggregate value for a group.

Figure 8 is *Eager Monotonic Aggregate Semi-naive Evaluation* (*EMSN*). The main idea with *EMSN* is that recursive rules with monotonic aggregates are evaluated fact-at-a-time while

```
1:  S := M;
2:  δS_A := getGroupRep(T_{E_A}(M));
3:  δS := T_{E_N}(M);
4:  S := S ∪ δS ∪ δS_A;
5:  while δS ≠ ∅ and δS_A ≠ ∅ do
6:      for all fact ∈ δS_A do
7:          while (newfact := T_{R_A}(getCurrent(fact))) do
8:              δS'_A := δS'_A ∪ {newfact};
9:      δS'_A := getGroupRep(δS'_A);
10:     δS' := T_{R_N}(δS) − S;
11:     S := S ∪ δS' ∪ δS'_A;
12:     δS := δS'; δS_A := δS'_A;
13: return S;
```

Fig. 8. *Eager Monotonic Aggregate Semi-naive Evaluation* Sketch

the other rules are evaluated set-at-a-time like in *Semi-naive*. Monotonic aggregate rules are partitioned from the other rules and two sets of Immediate Consequence Operators (ICO) will be used. T_{E_A} and T_{R_A} are the ICO for the monotonic-aggregate exit and recursive rules, respectively, and T_{R_A} will be applied on one fact at a time. T_{E_N} and T_{R_N} are the ICO for the remaining exit and recursive rules, respectively. δS_A and $\delta S'_A$ contain facts obtained during the previous and current iteration, respectively for the monotonic-aggregate rules, while δS and $\delta S'$ contain facts obtained during the previous and current iteration, respectively, for the remaining rules. M is the initial model, S contains all facts obtained thus far. $fact$ and $newfact$ are facts from δS_A and from a single application of T_{R_A}, respectively.

Key points of Figure 8 are as follows. We use $getGroupRep()$ on the set produced by $T_{E_A}(M)$ to produce the initial δS_A (line 2). $getGroupRep()$, produces a set containing only one aggregate predicate fact per group. For example, given spaths(a, b, 2) and spaths(a, b, 1), $getGroupRep()$ would produce a set with either one representing (a, b) for spaths. Then, T_{E_N} is applied to M to produce the initial δS (line 3). Once in the recursion, individually, each fact in δS_A is used to retrieve its group's current fact $getCurrent(fact)$ from the aggregate relation, which T_{R_A} is then applied to (line 7). Upon successful derivation by T_{R_A}, $newfact$ is added to $\delta S'_A$ (line 8). After all facts in δS_A are used, $getGroupRep$ is applied to $\delta S'_A$ to get the δ facts for aggregates for the next iteration. Then, T_{R_N} (remaining rules) is applied to δS which, after duplicates are eliminated, produces $\delta S'$, the set of facts to be used in the next iteration (line 10). This process repeats until no new facts are produced during an iteration.

Example 1 *r2* evaluation with *EMSN*
spaths(a, c, 2) $\leftarrow$ spaths(a, b, 1), edge(b, c, 1), 2 = 1+1.
spaths(a, c, 3) $\leftarrow$ **spaths(a, c, 2)**, edge(c, d, 1), 3 = 2+1.

Fig. 9. *EMSN* Fact-at-a-time Efficiency

Now, consider the same scenario from Figure 7, but this time using *EMSN* from Figure 8. In Figure 9, now after spaths(a, c, 2) is produced, it is immediately used in the next derivation. This results in a successful derivation, and one iteration earlier than with *Semi-naive* or *MASN*. Moreover, with spaths(a, c, 2) now derived, we can ignore spaths(a, c, 3) as it will not lead to a final result (*Monotonic Optimization*).

Discussion Since *EMSN* evaluates the ICO for recursive monotonic aggregate rules (T_{R_A}) on an individual fact, rather than on a set of facts, it can use facts immediately upon derivation. Although *EMSN* is based on *Semi-naive*, and therefore BFS, *EMSN* has depth-first search (DFS) characteristics. Like BFS, *EMSN* still uses a level-at-a-time (iteration) approach guided by facts in δS and δS_A that were derived during the previous iteration. However, because *EMSN* uses the most recent aggregate value for the group the fact belongs to, regardless of when the value was computed, *EMSN* can evaluate deeper than a single level of the search space during an iteration of evaluation. The result is higher (mmax) or lower (mmin) aggregate values being derived earlier in evaluation,

which in turn prunes the search space to avoid derivation of facts that will not result in final values.

V. Implementation

In this section, we present details of $DeAL$'s `mmin` and `mmax` aggregate implementation.

A. System Overview

$DeALS$ is an interpreted Datalog system with three main components — the compiler, the interpreter and the storage manager. Monotonic aggregate rules are supported by the compiler with an aggregate rewriting approach based on techniques from [27]. The interpreter uses tuple-at-a-time pipelining, evaluating rule bodies in a left-to-right fashion with backtracking. From the binding pass analysis, $DeALS$ determines index and cursor selection. For instance, from Example 1, in *r1*, both `X` and `Y` are free, so `edge` will be scanned, while in *r2*, `spaths` will be scanned and `edge` will be indexed on `Z` (first argument) because `Z` is bound by `spaths`.

B. Storage Manager Overview

$DeAL$'s storage manager provides support for main memory storage and indexing for predicate relations. $DeALS$ supports several B+Tree data structure variants for tuple storage. B+Trees stores fixed-size keys in internal and leaf nodes and fixed-size non-key attributes in leaf nodes. Leaf nodes have pointers to their right neighbors for fast linear scan of the tree. Through testing we determined our implementations perform best on average using a linear key search at both internal and leaf nodes with 256 bytes allocated for keys. $DeALS$ also supports an *Unordered Heap TupleStore* (*UHT*) where tuples are stored as fixed-size entries in insertion order in large byte arrays. *UHT* can be given multiple B+Tree indexes, which they remain synchronized with at all times. *UHT* enable a highwatermark approach for *Semi-naive evaluation* where each iteration is a contiguous range of tuples.

B+Tree Aggregators Early experiments found aggregation with *UHT* lacking in execution time performance. The *B+Tree Aggregator TupleStore* (*BAT*) is a B+Tree design optimized for pipelined aggregation in recursive queries that provides both good read and write performance. *BAT* store fixed-size keys in internal and leaf nodes and fixed-size aggregate values in leaf nodes. Keys are unique and only one aggregate value per key is maintained. Leaf nodes have pointers to their right neighbors and linear key search is used in both internal and leaf nodes. In a *BAT*, aggregation is performed in the leaves, therefore only one search is needed to retrieve the previous value, compare it with the new value and perform the update.

Unlike with *UHT*, facts in our B+Trees are not easily tracked by reference or range because of node splitting. In recursive queries, after a modification is made in a leaf, *BAT* will insert the fact's key into a second B+Tree[3], which maintains the set of keys to process for the next iteration ($\delta S'_A$ in Figure 8). *EMSN* will scan this second B+Tree using a cursor, and for each key, retrieve the fact from the *BAT*, which contains the current aggregate value for the key.

C. `mmax` and `mmin` Implementation

The `mmax` and `mmin` implementation tracks the greatest (`mmax`) or least (`mmin`) value computed for each group where each group has one tuple in the TupleStore. We use a single relation schema with one column for each of the predicate's *group-by argument* and a column for the aggregate value. Specifically, *BAT* keys are the group-by arguments with the aggregate value stored in the leaf. *UHT* are indexed (B+Tree) on the group-by arguments. For instance, `spaths` in Example 1 would use *BAT* with keys (`X`, `Y`) and each `X`, `Y` would be stored with its current value (`D`) in a leaf node.

D. Operational Optimizations

No Recursive Relation Storage Due to the *Monotonic Optimization*, we only need to maintain a single fact per group and when a new value for the group is successfully derived, we overwrite the previous value. If the recursive predicate and monotonic aggregate use separate stores, with *EMSN* and pipelining, the result is the recursive relation storage is merely being synchronized with the aggregate relation storage. Therefore, we do not allocate the recursive predicate storage, and instead have it read from the monotonic aggregate storage.

Final Results via Monotonic Aggregate Since the monotonic aggregate maintains the value for each group in its TupleStore, when a fixpoint is reached, its TupleStore contains the final results. For instance, instead of evaluating *r3* in Example 1 the recursion is materialized by the system, as it would have been by the stratified aggregate, and $DeALS$ retrieve the final values from the monotonic aggregate's TupleStore.

VI. mmin & mmax Performance Analysis

All experiments on synthetic graphs were run on a machine running Ubuntu 14.04 LTS 64-bit with an i7-4770 CPU and 32GB memory. The experiments on real-life graphs were run on a machine running Ubuntu 12.04 LTS 64-bit with four AMD Opteron 6376 CPUs and 256GB memory. The memory utilization is collected by the Linux `time` command. Execution time and memory utilization are calculated by performing the same experiment five times, discarding the highest and lowest values, and taking the average of the remaining three values. All the experiments on systems written in Java were run using Java 1.8.0 except for SociaLite (0.8.1) which doesn't support Java 1.8.0. The experiments for SociaLite were run using Java 1.7.0.

Datasets An n-vertex graph used in experiments has integer vertex labels ranging from 0 to $n-1$. We used three kinds of synthetic graphs — 1) directed acyclic graphs (DAGs), generated by connecting each pair of vertices i and j ($i < j$) with (edge) probability p; 2) random graphs, generated by connecting each pair of vertices with (edge) probability p; 3) scale-free graphs, generated using GTgraph[4]. The graphs are *shuffled* after generation where one random permutation is

[3]We use a B+Tree because the keys can be scanned in order, which can benefit *EMSN*.

[4]GTgraph, http://www.cse.psu.edu/~madduri/software/GTgraph/.

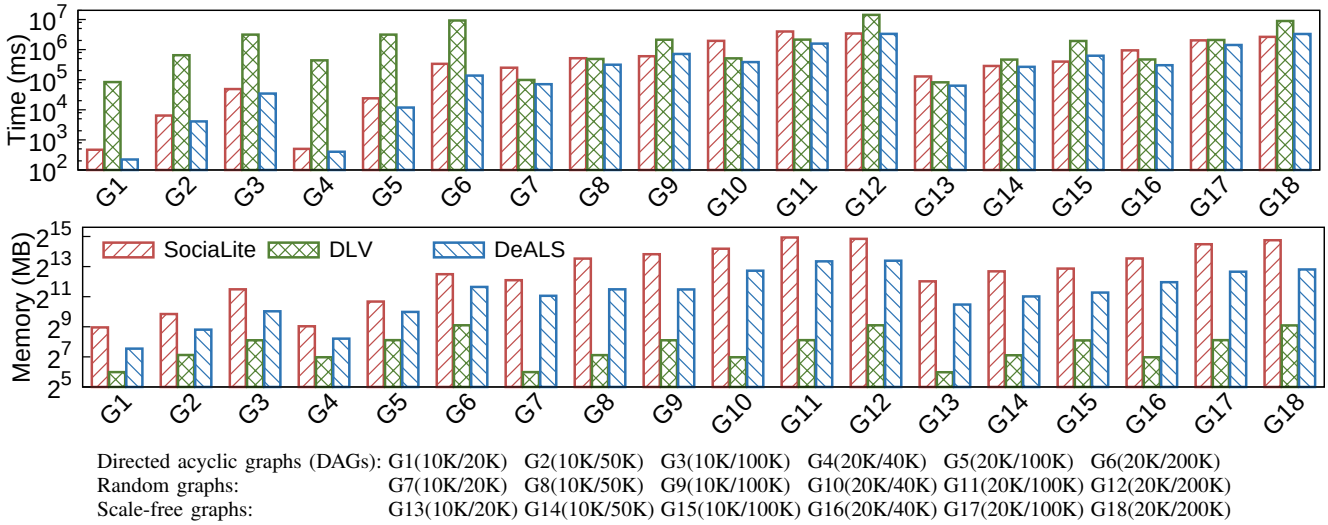


Fig. 10. Execution time and memory utilization of all-pairs shortest paths query on synthetic graphs.

applied to the vertex labels and another random permutation is applied to the edges. The real-life graphs are not shuffled. We only relabeled the graphs whose vertex labels are beyond the range of [0, $n-1$] while maintaining the original edge order. A text description such as "10K/20K" indicates the graph has 10,000 vertices and 20,000 edges.

Configuration *BAT* and B+Tree indexes for *UHT* were configured with 256 bytes allocated for keys in each node (internal and leaf). Other than experiments in Section VI-C, $DeALS$ used *EMSN* with *BAT*.

A. Datalog Language Implementation Comparison

$DeALS$ is a sequential, main memory Java implementation. We compare its execution time and memory utilization on the shortest path query against the other three Datalog language implementations supporting aggregates in recursion — 1) the DLV system[5], which is the state-of-the-art implementation of disjunctive logic programming; 2) the commercial LogicBlox system, which recently included support for aggregates in recursion using *staged partial fixpoint* semantics[6]. Based on an analysis of log files during query execution, we determined LogicBlox indeed uses an approach akin to *Semi-naive* and only uses new shortest paths found in the current iteration in derivations during the next iteration; 3) the SociaLite[7] graph query language [2] which supports a left-linear recursive expression of the shortest path query and evaluates it using *Semi-naive*. SociaLite efficiently evaluates single-source shortest paths (SSSP) and queries of a similar form using an approach with Dijkstra's algorithm-like efficiency [2]. We compared with SociaLite version 0.8.1 because it had the best execution time performance on sequential queries for SociaLite versions available to us.

[5]DLV with recursive aggregates support, http://www.dbai.tuwien.ac.at/proj/dlv/dlvRecAggr/.

[6]LogicBlox 4 migration guide, https://download.logicblox.com/wp-content/uploads/2014/05/LB-MigrationGuide-40.pdf.

[7]https://sites.google.com/site/socialitelang/

Synthetic graphs. Figure 10 shows the results of the APSP query on synthetic graphs with random integer edge costs between 1-100. We tested two versions of LogicBlox — 3.10.15 and 4.1.3. The former version doesn't supports aggregates in recursion. Although we can express the APSP query in a stratified program, it only terminates on DAGs, and only the computation on G1(0.286s) and G2(18.328s) finishes within 24 hours. The latter version supports aggregates in recursion. Still, only the computation on G1(4.809s), G2(6.697m), G7(4.774h) and G13(3.977h) finishes within 24 hours. We don't report the results of LogicBlox in the figure and the remaining experiments.

Among the 18 graphs, SociaLite is the fastest on two graphs, and $DeALS$ is the fastest on the remaining 16 graphs. $DeALS$ is more than two times faster than SociaLite on sparse graphs that the average degree of each vertex is only two (e.g., G7, G10 and G16). This advantage reduces as the average degree increases from two to ten. The main reason for this change is due to the different designs between $DeALS$ and SociaLite. SociaLite uses an array of hash tables with an initial capacity of around 1,000 entries to maintain the delta relations, whereas $DeALS$ uses a B+Tree. The initialization cost of a hash table is higher than that of a B+Tree, while the cost of accessing a hash table is lower than that of a B+Tree. For graphs with small average degree, the initialization cost may account for a large percentage of the execution time, thus $DeALS$ is faster than SociaLite. The impact of the initialization cost reduces as the average degree increases, and thus SociaLite is faster than $DeALS$ on denser graphs. But this speed comes at the cost of higher memory utilization. SociaLite uses more than two times memory than $DeALS$ on all 18 graphs. Although the C-based system DLV has a significantly lower memory utilization than Java-base systems $DeALS$ and SociaLite, it is extremely slow comparing with both $DeALS$ and SociaLite on DAGs. To sum up, $DeALS$ achieves the best speed and memory trade-off on sparse graphs among the three compared systems.

Real-life graphs. Table I shows the results of the APSP query on three real-life graphs from the Stanford large network dataset collection[8]. These graphs don't have edge costs. We assigned unit cost to each edge. The results are similar to that of synthetic graphs — $DeALS$ is the fastest while DLV has the lowest memory utilization. These results suggest that, the B+Tree-based design (low initialization cost) adopted by $DeALS$ is more favorable than the hash table-based design on real-life workloads.

TABLE I
EXECUTION TIME AND MEMORY UTILIZATION OF ALL-PAIRS SHORTEST PATHS QUERY ON REAL-LIFE GRAPHS.

	HepTh			Gnutella			Slashdot		
	S1	S2	S3	S1	S2	S3	S1	S2	S3
Time(h)	0.98	17.72	0.39	13.31	4.69	0.49	>24.00	>24.00	2.72
Mem(GB)	12.76	0.99	7.19	41.57	0.48	23.46	>89.59	>1.06	64.70

S1, S2, S3 represent SociaLite, DLV and $DeALS$ respectively.
HepTh(28K/353K): High-energy physics theory citation network.
Gnutella(63K/148K): Gnutella peer-to-peer network.
Slashdot(82K/549K): Slashdot social network.

SSSP on real-life graphs. Figure 11 shows the results of the SSSP query on five real-life graphs from the USA road networks datasets[9]. For each graph, we evaluated the SSSP query on ten randomly selected vertices, and we report the min / (geometric) average / max execution time in the form of error bars. The only time we can get for DLV includes the time for loading the graph, evaluating the query and outputting the result, in which the query evaluation time only accounts for a small percentage. Thus, the time for DLV is less informative for this query, and we only report the results for SociaLite and $DeALS$. SociaLite generates a Java program that evaluates the query using the Dijkstra's algorithm. The generated code achieves more than one order of magnitude speedup comparing to LogicBlox [2]. However, our interpreted system $DeALS$ is faster than code-generated SociaLite for the SSSP query on the road network graphs reported in Figure 11. This result is not surprising in the sense that *EMSN* optimizes *Semi-naive*, and the Bellman-Ford algorithm (equivalent to *Semi-naive*) usually yields comparable performance with the Dijkstra's algorithm on large sparse graphs.

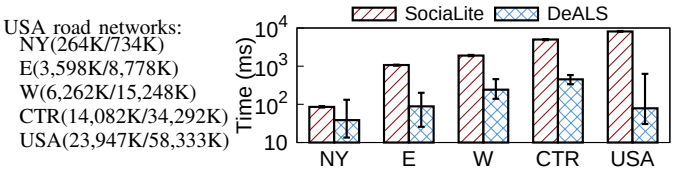


Fig. 11. Execution time of single-source shortest paths query.

B. Statistical Analysis of Evaluation Methods

To provide a characterization of the relative performance of *EMSN* compared to *Semi-naive*, we performed an analysis over sets of graphs using the statistical estimation technique from [28]. Statistics calculated[10] are 1) total number of facts derived by the recursive predicate (*derived facts*) and 2) total aggregate size of δS across all iterations (δ *facts*). These two statistics help to quantify the amount of computation the evaluation method must perform. For each statistic and each vertex number/edge probability combination, after each run of the program, the statistic is included in the average ($\bar{m}$) until a statistically significant number (30) of different graphs has been used AND $\bar{m}$ is within 5% error with 95% confidence[11].

We performed the analysis using *EMSN* and *Semi-naive* implementation of APSP. We used randomly generated DAGs and random graphs with edge probability between 0.1 and 0.9 (increments of 0.1) and random integer edge cost between 1-50. *EMSN* and *Semi-naive* used the same sequence of graphs. On DAGs, *Semi-naive* derived between 3-11% more derived and δ facts and on random graphs between 13-18% more derived and δ facts than *EMSN*, respectively. We interpret these results to be validation of the increased efficiency of *EMSN*'s fact-at-a-time approach for monotonic aggregate evaluation over *Semi-naive*.

C. Storage Manager Evaluation

This experiment shows i) how *EMSN* performs relative to *MASN* and ii) how *BAT* performs relative to *UHT*. We evaluated APSP on synthetic graphs G1, G2, G7, G8, G13 and G14. The (geometric) average execution time and memory utilization over the six graphs are displayed in Table II. These results help to explain the benefit of using *EMSN* instead of *MASN*. With the same storage configuration *EMSN* had a lower average execution time than *MASN* by 13%. A much more noticeable difference in execution time performance was observed between using *BAT* vs. *UHT* for *EMSN*. Not only did *BAT* require far less memory for *EMSN*, roughly 25% of *UHT* memory utilization, but *BAT* was 2.6 times faster.

TABLE II
STORAGE MANAGER EVALUATION BY EVALUATION TYPE

Evaluation Type	Storage Configuration	Time (s)	Memory (GB)
MASN	*UHT* w/ B+Tree index	77.743	4.087
EMSN	*UHT* w/ B+Tree index	68.927	4.131
EMSN	*BAT*	26.450	1.048

VII. MONOTONIC COUNT AND SUM

$DeALS$ also efficiently supports monotonic count and sum aggregates allowing $DeAL$ to support many exciting applications.

A. `mcount` & `msum` Monotonic Aggregates

An `mcount` or `msum` monotonic aggregate rule has the form:

$$\texttt{p}(\texttt{K}_1, \ldots, \texttt{K}_\texttt{m}, \texttt{aggr}\langle(\texttt{T}, \texttt{P}_\texttt{T})\rangle) \leftarrow \texttt{Rule Body}.$$

$\texttt{K}_1, \ldots, \texttt{K}_\texttt{m}$ are the zero or more *group-by arguments* ($\overline{\texttt{K}}$). $\texttt{aggr} \in \{\texttt{mcount}, \texttt{msum}\}$ is the *monotonic aggregate*. $(\texttt{T}, \texttt{P}_\texttt{T})$

[8]Stanford large network dataset, http://snap.stanford.edu/data/index.html.

[9]USA road networks, http://www.dis.uniroma1.it/challenge9/download.shtml.

[10]For these statistics, we assume a normal distribution ($N(\mu, \sigma^2)$, with mean μ and variance σ^2).

[11]As in [28], $\bar{m}$ is accepted when $\epsilon < (0.05 * \bar{m})$, where $\epsilon = (1.96 * \sigma)/\sqrt{k}$. σ is standard deviation. k is the number of graphs used. 1.96, from the tables for standard normal distribution for 0.975, gives the 95% confidence coefficient.

is the *aggregate term* pair where T is a variable passed in from the body and P_T is a constant or a variable passed from the body indicating the partial count/sum contributed by T.

As with mmin and mmax, the mcount and msum aggregates are monotonic w.r.t. set-containment and can be used in recursive rules, and memorize the most recently computed value. When viewed as a sequence, the values produced by mcount and msum is monotonic. mcount and msum are aggregate functions that map an input set or multiset, we will call G, to an output set, we will call D. Elements $g \in G$ have the form (J, N_J), where N_J indicates the partial count/sum contributed by a J. Note, J maps to T and N_J maps to P_T in the definition of the aggregate term.

Now, given G, for each element $g \in G$, if $N_J > N_{J_{prev}}$, where $N_{J_{prev}}$ is the previous count (sum) for J or 0 if no previous count (sum) for J exists, mcount (msum) computes the count (sum) for G by summing all maximum partial counts (sums) N_J for all J. Since only the maximum N_J for each J is aggregated, no double counting (summing) occurs. Lastly, msum will only compute with positive numbers, thereby ensuring its monotonicity.

Running Example Example 2 is the $DeAL$ program to count the paths between pairs of vertices in an acyclic graph. This program is not expressible with Datalog with stratified aggregation [8]. We will use Example 2 as our running example for mcount to explain counting in $DeAL$.

Example 2: Counting Paths in a DAG

r1. $\texttt{cpaths}(\texttt{X},\texttt{Y},\texttt{mcount}\langle(\texttt{X},1)\rangle) \leftarrow \texttt{edge}(\texttt{X},\texttt{Y})$.
r2. $\texttt{cpaths}(\texttt{X},\texttt{Y},\texttt{mcount}\langle(\texttt{Z},\texttt{C})\rangle) \leftarrow \texttt{cpaths}(\texttt{X},\texttt{Z},\texttt{C}),\texttt{edge}(\texttt{Z},\texttt{Y})$.
r3. $\texttt{countpaths}(\texttt{X},\texttt{Y},\texttt{max}\langle\texttt{C}\rangle) \leftarrow \texttt{cpaths}(\texttt{X},\texttt{Y},\texttt{C})$.

In Example 2, *r1* counts each edge as one path between its vertices. In *r2*, any edge(Z, Y) that extends from a computed path count cpath(X, Z, C) establishes there are C distinct paths from X to Y through Z. The $\texttt{mcount}\langle(\texttt{Z},\texttt{C})\rangle$ aggregate in the head sums the count of paths from X to Y through every Z to produce the count from X to Y. Lastly, *r3* indicates only the maximum count for each path X, Y in cpaths is desired. As explained in Section V-D, *r3* does not have to be evaluated.

As an example of the interval semantics - if *r2* in Example 2 produced $\texttt{cpaths}(\texttt{a},\texttt{b},3)$ and then $\texttt{cpaths}(\texttt{a},\texttt{b},4)$, we cannot sum the aggregate values to get a new count for $(\texttt{a},\texttt{b})$. Instead, with the counts for $\texttt{cpaths}(\texttt{a},\texttt{b},3)$ and $\texttt{cpaths}(\texttt{a},\texttt{b},4)$ represented with $[1,3]$ and $[1,4]$, respectively, $[1,3] \cup [1,4] = max(3,4) = 4$. Thus $\texttt{cpaths}(\texttt{a},\texttt{b},4)$ represents $(\texttt{a},\texttt{b})$'s count.

B. mcount *by Example*

To further explain the mcount aggregate, we walk through an evaluation of Counting Paths (Example 2) using *EMSN*. The edge facts in Figure 12 is the example dataset. First, *r1* in Example 2 is evaluated and results in the six cpaths facts as shown in the Figure 13. Each cpaths fact has a count of 1 indicating one path between each pair of vertices connected by edge facts. The partial count is memorized (recall (J, N_J) discussed above) for each group (e.g. $(\texttt{a},\texttt{b})$), also displayed in the right column of Figure 13. For example, in the first derivation, J=a, N_J=1, $(\texttt{a},1)$ is memorized for $(\texttt{a},\texttt{b})$.

Facts
$\texttt{edge}(\texttt{a},\texttt{b})$.
$\texttt{edge}(\texttt{a},\texttt{c})$.
$\texttt{edge}(\texttt{a},\texttt{d})$.
$\texttt{edge}(\texttt{b},\texttt{c})$.
$\texttt{edge}(\texttt{b},\texttt{d})$.
$\texttt{edge}(\texttt{c},\texttt{d})$.

Fig. 12. Facts

r1 Successful Derivations	Partial Count
$\texttt{cpaths}(\texttt{a},\texttt{b},1) \leftarrow \texttt{edge}(\texttt{a},\texttt{b})$.	$(\texttt{a},1)$ for $(\texttt{a},\texttt{b})$
$\texttt{cpaths}(\texttt{a},\texttt{c},1) \leftarrow \texttt{edge}(\texttt{a},\texttt{c})$.	$(\texttt{a},1)$ for $(\texttt{a},\texttt{c})$
$\texttt{cpaths}(\texttt{a},\texttt{d},1) \leftarrow \texttt{edge}(\texttt{a},\texttt{d})$.	$(\texttt{a},1)$ for $(\texttt{a},\texttt{d})$*
$\texttt{cpaths}(\texttt{b},\texttt{c},1) \leftarrow \texttt{edge}(\texttt{b},\texttt{c})$.	$(\texttt{b},1)$ for $(\texttt{b},\texttt{c})$
$\texttt{cpaths}(\texttt{b},\texttt{d},1) \leftarrow \texttt{edge}(\texttt{b},\texttt{d})$.	$(\texttt{b},1)$ for $(\texttt{b},\texttt{d})$
$\texttt{cpaths}(\texttt{c},\texttt{d},1) \leftarrow \texttt{edge}(\texttt{c},\texttt{d})$.	$(\texttt{c},1)$ for $(\texttt{c},\texttt{d})$

Fig. 13. Example 2 *r1* evaluation

r2 Successful Derivations	Partial Count
$\texttt{cpaths}(\texttt{a},\texttt{c},2) \leftarrow \texttt{cpaths}(\texttt{a},\texttt{b},1),\texttt{edge}(\texttt{b},\texttt{c})$.	$(\texttt{b},1)$ for $(\texttt{a},\texttt{c})$
$\texttt{cpaths}(\texttt{a},\texttt{d},2) \leftarrow \texttt{cpaths}(\texttt{a},\texttt{b},1),\texttt{edge}(\texttt{b},\texttt{d})$.	$(\texttt{b},1)$ for $(\texttt{a},\texttt{d})$*
$\texttt{cpaths}(\texttt{a},\texttt{d},4) \leftarrow \texttt{cpaths}(\texttt{a},\texttt{c},2),\texttt{edge}(\texttt{c},\texttt{d})$.	$(\texttt{c},2)$ for $(\texttt{a},\texttt{d})$*
$\texttt{cpaths}(\texttt{b},\texttt{d},2) \leftarrow \texttt{cpaths}(\texttt{b},\texttt{c},1),\texttt{edge}(\texttt{c},\texttt{d})$.	$(\texttt{c},1)$ for $(\texttt{b},\texttt{d})$

Fig. 14. Example 2 *r2* evaluation, 1st iteration

EMSN evaluates the recursive *r2* rule from Example 2 using the cpaths derived by *r1*. Figure 14 shows the successful derivations performed by *r2*. Four cpaths facts are derived bringing the total number of cpaths facts to ten. Note the derivation of $\texttt{cpaths}(\texttt{a},\texttt{d},2)$ from joining $\texttt{cpaths}(\texttt{a},\texttt{b},1)$ and $\texttt{edge}(\texttt{b},\texttt{d})$. It represents a count of two for $(\texttt{a},\texttt{d})$, even though the rule body contributed only one path count. However, looking at the *-ed entry in Figure 13, we see a partial count of $(\texttt{a},1)$ towards $(\texttt{a},\texttt{d})$ was accrued during evaluation of *r1*. Therefore, when computing the new count for $(\texttt{a},\texttt{d})$, $(\texttt{a},1)$ and the newly derived $(\texttt{b},1)$ are summed to result in $\texttt{cpaths}(\texttt{a},\texttt{d},2)$. Next, we observe the benefits of using *EMSN* with the derivation of $\texttt{cpaths}(\texttt{a},\texttt{d},4)$. Since $\texttt{cpaths}(\texttt{a},\texttt{c},2)$ existed even though it was derived this iteration, it was used and successfully joined with $\texttt{edge}(\texttt{c},\texttt{d})$. Then the partial counts for $(\texttt{a},\texttt{d})$, which are $(\texttt{a},1)$, $(\texttt{b},1)$, and $(\texttt{c},2)$, are summed to produce $\texttt{cpaths}(\texttt{a},\texttt{d},4)$. With *r2* exhausted, a fixpoint is reached and we have our result.

VIII. Counting Implementation

In this section, we present details of $DeAL$'s monotonic count and sum aggregate implementation. We use definitions from Section VII (e.g. G). Note, G is a single group produced from the implicit *group-by* for a distinct assignment of $\overline{\texttt{K}}$, the zero or more *group-by* arguments.

A. mcount *and* msum *Implementation*

Although we use mcount to present our efficient count/sum technique, it is easily generalizable to msum.

We present an efficient approach for count and sum using delta-maintenance (Δ-*Maintenance*) techniques. Recalling our explanation for mcountin Section VII, given a new partial count $N_J > N_{J_{prev}}$, mcount will sum all maximum partial counts to compute the new total count for group G. However, rather than recompute the count, we can instead use Δ-*Maintenance* to increase N (the current total count for G) by $N_J - N_{J_{prev}}$ and put the updated count, now the total current count for G, into output set D. This produces the same result as if the maximum partial count N_J for all J are summed

to produce the total count N for G, however avoids the re-summation of all N_J with each change in a N_J. This requires memorizing both N for G and N_J for all J.

Storage Design Table III displays storage designs we investigated for `mcount` and `msum`. Here we use `N` to indicate the current count/sum for the group-by arguments ($\overline{\texttt{K}}$). As in Section VII-A, each `T` contributes a partial count/sum $\texttt{P}_\texttt{T}$ for a distinct assignment of $\overline{\texttt{K}}$.

Double uses two relations, one relation $(\overline{\texttt{K}}, \texttt{N})$ indexed on $(\overline{\texttt{K}})$ to store tuples containing the group's total aggregate value and a second relation $(\overline{\texttt{K}}, \texttt{T}, \texttt{N}_\texttt{J})$ indexed on $(\overline{\texttt{K}}, \texttt{T})$ to store the partial count $\texttt{N}_\texttt{J}$ for each distinct assignment of $(\overline{\texttt{K}}, \texttt{T})$. Early testing showed the *Double* using *UHT* without Δ-*Maintenance* to take 2-5 times longer to execute then when using Δ-*Maintenance*.

Next, we investigated designs using *KeyValue* type columns as a more efficient way of managing $(\texttt{T}, \texttt{P}_\texttt{T})$ pairs. We developed three single relation designs $(\overline{\texttt{K}}, \texttt{N}, \mathit{KeyValue}[(\texttt{T}, \texttt{P}_\texttt{T})])$, where `N` is the total count for $\overline{\texttt{K}}$ and $\mathit{KeyValue}[(\texttt{T}, \texttt{P}_\texttt{T})]$ is a reference to the tuple's own *KeyValue* type data structure. The relation is indexed on $\overline{\texttt{K}}$ and each group has a single tuple. The *KeyValue* types each represent a different retrieval time complexity; a *List* ($O(n)$) type, a *B+Tree* ($O(log(n))$) type, and a *Hashtable*($O(1)$) type. *Hashtable* is based on Linear Hashing and stores the hashed key in the bucket to avoid rehashing. *B+Tree* stores keys (`T`) in internal and leaf nodes and non-key attributes ($\texttt{P}_\texttt{T}$) in leaf nodes, and uses linear search. Lastly, *List* stores $(\texttt{T}, \texttt{P}_\texttt{T})$ pairs ordered by `T` and uses a linear search. These are main memory structures, so designs attempt to limit the number of objects (e.g. *List* uses byte arrays). *KeyValue* designs use Δ-*Maintenance*.

Recall our options for tuple storage from Section V-B. For the designs shown in Table III, *DeALS* supports *List*, *HashTable* and *B+Tree* with *BAT* and all designs with *UHT* indexed as shown. For example, using *r1*, *r2* from Example 2, with *B+Tree*, the *BAT* would have `X, Y` as keys and the each entry in a leaf would have the current total count `N` and a reference to a second B+Tree to store $(\texttt{T}, \texttt{P}_\texttt{T})$ pairs. This design is essentially a B+Tree of B+Trees.

IX. Counting Performance Analysis

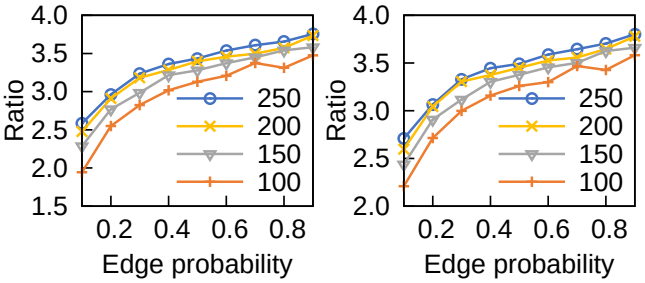
Configuration B+Trees, *BAT* and B+Trees in the *B+Tree* design for `mcount` and `msum` were configured with 256 bytes allocated for keys in each node (internal and leaf). The *Hashtable* design for `mcount` and `msum` used a directory and segment size of 256, 16 initial buckets and split policy of 10.

TABLE III
Schema Designs

Name	Schema	Indexes
Double	$(\overline{\texttt{K}}, \texttt{N})$ | $(\overline{\texttt{K}}, \texttt{T}, \texttt{P}_\texttt{T})$	$\overline{\texttt{K}}$ | $(\overline{\texttt{K}}, \texttt{T})$
List	$(\overline{\texttt{K}}, \texttt{N}, \mathit{List}[(\texttt{T}, \texttt{P}_\texttt{T})])$	$\overline{\texttt{K}}$
B+Tree	$(\overline{\texttt{K}}, \texttt{N}, \mathit{B+Tree}[(\texttt{T}, \texttt{P}_\texttt{T})])$	$\overline{\texttt{K}}$
Hashtable	$(\overline{\texttt{K}}, \texttt{N}, \mathit{Hashtable}[(\texttt{T}, \texttt{P}_\texttt{T})])$	$\overline{\texttt{K}}$

A. Statistical Analysis of Evaluation Methods

We also performed the statistical analysis described in Section VI-B comparing Counting Paths (Example 2) using *EMSN* with a *Semi-naive* implementation of Counting Paths. The experiment used randomly generated DAGs of 100-250 vertices (increments of 50) and edge probability between 0.1 and 0.9 (increments of 0.1). *EMSN* and *Semi-naive* used the same sequence of graphs. Figures 15(a) and 15(b) show the results of the analysis.



(a) Derived Facts (b) δ Facts

Fig. 15. Ratio *Semi-naive*/*EMSN* Derivations - Counting Paths

Each point on a line represents the ratio of *Semi-naive* to *EMSN* for number of facts derived (Figure 15(a)) or number of δ facts (Figure 15(b)) for the size of the graph indicated by the line and edge probability indicated by the x-axis. For example, in Figure 15(b), *Semi-naive* produces greater than three times as many δ facts as *EMSN* for graphs of size 200 and 250 vertices starting around 0.2 (20%) edge probability. Figures 15(a) and 15(b) show that for our test graphs, as the edge probability increases, we observe Counting Paths using *Semi-naive* requires 1.94-3.48 times as many derivations than Counting Paths using *EMSN*. We attributed the increase in ratio as edge probability increases to the increased edge density allowing *EMSN* greater opportunity to use facts earlier in derivations and thus prune the search space. These results validate *EMSN*'s efficiency for evaluating recursive programs with count and sum monotonic aggregates.

B. Storage Design Evaluation

In this experiment, we tested how each of the storage designs presented in Section VIII-A would perform on DAGs. Figure 16 shows the (geometric) average execution time and memory utilization, along with minimum and maximum values, on 45 random 250-vertex DAGs (5 graphs for each edge probability from 0.1 to 0.9) for each design. Designs are shown from worst to best average execution time performance in left-to-right order in Figures 16.

Recall the descriptions from Sections V-B and VIII-A. D2 is the *Double* design as described in Table III executed using *UHT* with B+Tree indexes. D1 is *Double* using a B+Tree TupleStore for the $(\overline{\texttt{K}}, \texttt{T}, \texttt{P}_\texttt{T})$ relation[12]. D3-D5 and D6-D8 are *KeyValue* designs executed using *UHT* and *BAT*, respectively. Figure 16 shows D6-D8, the three *BAT KeyValue* type designs,

[12]The $(\overline{\texttt{K}}, \texttt{T}, \texttt{P}_\texttt{T})$ relation contains many times more tuples than *Double*'s $(\overline{\texttt{K}}, \texttt{N})$ relation (still *UHT*), which is why only it was made B+Tree here.

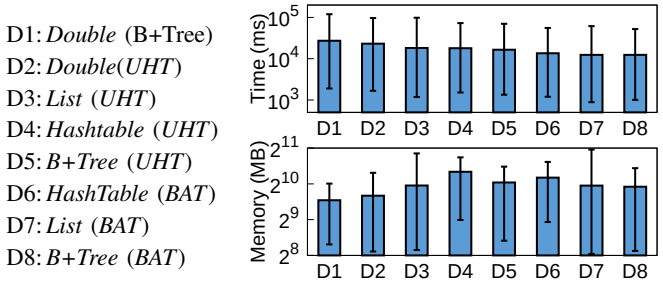


Fig. 16. `mcount` and `msum` Design Performance

as having had the best execution time performance. D7 and D8 had the lowest average execution time performance with *B+Tree* having better maximum (51s vs. 62s) execution time. Compared with D7, D8 had slightly better memory average utilization (969MB vs. 989MB) but lower maximum memory utilization (1.4GB vs. 2GB). Note, D1 and D2 had lowest average memory utilization but their average execution times were nearly twice that of D7 and D8. We conclude the *B+Tree* design using *BAT* shows the most promising among our designs for efficiently supporting `mcount` and `msum` as it balances good average execution time performance with average memory utilization.

C. Discussion

Finding other system with which to perform an experimental comparison for `mcount` and `msum` proved challenging. Support for count and sum aggregates that can be used in recursion is not as mature as that of min and max aggregates. Using LogicBlox version 4, we were able to express and execute the Counting Paths program, but we experienced the same slow performance as we did with APSP (Section VI-A). Using SociaLite, we were unable to execute the Counting Paths program, and BOM queries such as subparts explosion, produced results different from ground truth. Lastly, we were able to execute a version of the Counting Paths program in DLV, but again, the results were different from ground truth.

X. Example Programs

This section includes additional programs showing $DeAL$'s expressiveness and support for a variety of programs. More examples are found in [29] and on the $DeALS$ website[13].

Example 3: How many days until delivery?

$$r1.\ \texttt{delivery}(\texttt{Part}, \texttt{mmax}\langle\texttt{Days}\rangle) \leftarrow \texttt{basic}(\texttt{Part}, \texttt{Days}, \_).$$
$$r2.\ \texttt{delivery}(\texttt{Part}, \texttt{mmax}\langle\texttt{Days}\rangle) \leftarrow \texttt{assb}(\texttt{Part}, \texttt{Sub}, \_), \texttt{delivery}(\texttt{Sub}, \texttt{Days}).$$
$$r3.\ \texttt{actualDays}(\texttt{Part}, \texttt{max}\langle\texttt{Days}\rangle) \leftarrow \texttt{delivery}(\texttt{Part}, \texttt{Days}).$$

Example 4: What is the maximum cost of a part?

$$r1.\ \texttt{tcost}(\texttt{Part}, \texttt{msum}\langle(\texttt{Part}, \texttt{Cost})\rangle) \leftarrow \texttt{basic}(\texttt{Part}, \_, \texttt{Cost}).$$
$$r2.\ \texttt{tcost}(\texttt{Part}, \texttt{msum}\langle(\texttt{Sub}, \texttt{Cost})\rangle) \leftarrow \texttt{assb}(\texttt{Part}, \texttt{Sub}, \texttt{Quant}), \texttt{tcost}(\texttt{Sub}, \texttt{Scost}), \texttt{Cost} = \texttt{Scost} * \texttt{Quant}.$$
$$r3.\ \texttt{totcost}(\texttt{Part}, \texttt{max}\langle\texttt{Cost}\rangle) \leftarrow \texttt{tcost}(\texttt{Part}, \texttt{Cost}).$$

Example 3 and 4 are the Bill of Materials (BOM) program for finding the days required to deliver a part and program for computing the max cost of a part from the cost of its subparts, respectively. The `assb` predicate denotes each part's required subparts and number required and `basic` denotes the number of days for a part to be received and the part's cost.

Example 5: Viterbi Algorithm

$$r1.\ \texttt{calcV}(\texttt{0}, \texttt{X}, \texttt{mmax}\langle\texttt{L}\rangle) \leftarrow \texttt{s}(\texttt{0}, \texttt{EX}), \texttt{p}(\texttt{X}, \texttt{EX}, \texttt{L1}), \texttt{pi}(\texttt{X}, \texttt{L2}), \texttt{L} = \texttt{L1} * \texttt{L2}.$$
$$r2.\ \texttt{calcV}(\texttt{T}, \texttt{Y}, \texttt{mmax}\langle\texttt{L}\rangle) \leftarrow \texttt{s}(\texttt{T}, \texttt{EY}), \texttt{p}(\texttt{Y}, \texttt{EY}, \texttt{L1}), \texttt{T1} = \texttt{T} - \texttt{1}, \texttt{t}(\texttt{X}, \texttt{Y}, \texttt{L2}), \texttt{calcV}(\texttt{T1}, \texttt{X}, \texttt{L3}), \texttt{L} = \texttt{L1} * \texttt{L2} * \texttt{L3}.$$
$$r3.\ \texttt{viterbi}(\texttt{T}, \texttt{Y}, \texttt{max}\langle\texttt{L}\rangle) \leftarrow \texttt{calcV}(\texttt{T}, \texttt{Y}, \texttt{L}).$$

Example 5 is the Viterbi algorithm for hidden Markov models. `t` denotes the transition probability `L2` from state `X` to `Y`. `s` denotes the observed sequence of length `L+1`. `pi` denotes the likelihood `L2` that `X` is the initial state. `p` denotes the likelihood `L1` that state `X` (`Y`) emitted `EX` (`EY`). *r1* finds the most likely initial observation for each `X`. *r2* finds the most likely transition for observation `T` for each `Y`. Lastly, *r3* finds the max likelihood for each `T`, `Y`.

Example 6: Max Probability Path

$$r1.\ \texttt{reach}(\texttt{X}, \texttt{Y}, \texttt{mmax}\langle\texttt{P}\rangle) \leftarrow \texttt{net}(\texttt{X}, \texttt{Y}, \texttt{P}).$$
$$r2.\ \texttt{reach}(\texttt{X}, \texttt{Z}, \texttt{mmax}\langle\texttt{P}\rangle) \leftarrow \texttt{reach}(\texttt{X}, \texttt{Y}, \texttt{P1}), \texttt{reach}(\texttt{Y}, \texttt{Z}, \texttt{P2}), \texttt{P} = \texttt{P1} * \texttt{P2}.$$
$$r3.\ \texttt{maxP}(\texttt{X}, \texttt{Y}, \texttt{max}\langle\texttt{P}\rangle) \leftarrow \texttt{reach}(\texttt{X}, \texttt{Y}, \texttt{P}).$$

Example 6 is the non-linear program for computing the maximum probability path between two nodes in a network. $\texttt{net}(\texttt{X}, \texttt{Y}, \texttt{P})$ denotes the probability of reaching `Y` from `X` is `P`.

XI. Formal Semantics

So far we have worked with the operational semantics of our monotonic aggregates and shown how this is conducive to the expression of algorithms by programmers. While most users only need to work at this level, it is important that we also show how this coincides with the formal semantics discussed in those two Datalog^{FS} papers, inasmuch as properties such as least fixpoint, and stable models will follow from it.

We can start with the inspiring example by [9] where some people will come to the party for sure whereas others only join in when least three of their friends will come. The basic idea is that with `cntwillcome` each person watches the number of friends that `willcome` grow, and once that number reaches 3 our person join the party too. For that, rather than the final count used in [9] we can use the continuous count aggregate `mcount` that enumerates all the integers until the actual maximum, i.e. it returns the integer interval $[1, M]$ where M is the actual maximum.

Example 7: Who will come to the party?

$$r1.\ \texttt{willcome}(\texttt{X}) \leftarrow \texttt{sure}(\texttt{X}).$$
$$r2.\ \texttt{willcome}(\texttt{Y}) \leftarrow \texttt{cntwillcome}(\texttt{Y}, \texttt{N}), \texttt{N} \geq \texttt{3}.$$
$$r3.\ \texttt{cntwillcome}(\texttt{Y}, \texttt{mcount}\langle\texttt{X}\rangle) \leftarrow \texttt{friend}(\texttt{Y}, \texttt{X}), \texttt{willcome}(\texttt{X}).$$

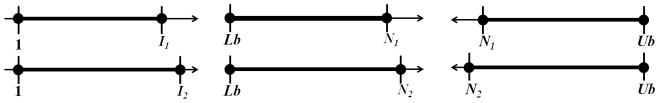
The use of `mcount` over `count` is here justified on the ground of performance, since there is no point in counting all the friends of popular people, if only three are required. Even more important is that while `count` is non-monotonic (unless we use the special lattices suggested by [9]), `mcount` is monotonic in the lattice of set containment used by the

[13] $DeALS$ website, http://wis.cs.ucla.edu/deals.

standard Datalog. So no ad hoc semantic extension is needed and concepts and techiques such as magic sets, perfect models and stable models can be immediately generalized to programs with `mcount`.

A. $DeAL$ Interval Semantics

The lessons learned with `mcount`, tells us that we can derive the monotonic counterpart of an aggregate by simply assuming that it produces an interval of integer values, rather than just one value. In the following we (i) apply this idea to max and min to obtain `mmax` and `mmin`, and then (ii) generalize these monotonic aggregates to arbitrary numbers, and show that the least fixpoint computation under this formal interval-based semantics can be implemented using the *Semi-naive* semantics use in Section III, under general conditions that hold for all our examples. Because of space limitations the discussion is kept at an informal level: formal proofs are given in [17], [18].



(a) Datalog^{FS} Interval Semantics (b) Arbitrary Numbers (c) Minimum Cost Semantics

Fig. 17. $DeAL$ Interval Semantics

+/- Rational Numbers Computations on numbers in mantissa * exponent representation *tantamount to integer arithmetic* on their numerators over a fixed large denominator. For floating point representations for instance, the smallest value of exponent supported could be -95, whereby every number can be viewed as the integer numerator of a rational number with denominator 10^{95}. However, because of the limited length of the mantissa, this numerator will often be rounded-off—a monotonic operation (see Section XI-B) that preserves the existence of a least-fixpoint semantics for our programs [17]. Thus floating point type representation in programming languages can be used without violating monotonicity [17], [18].

Now, say Lb represents the lower bound for all negative numbers supported by the system architecture. We can use the interval $[Lb, N]$ to represent any number regardless of its sign. The result of unioning the sets representing these numbers is a set representing the max, independent of whether this is positive or negative. Using Figure 17(b) as an example, with N_1 and N_2 represented by $[Lb, N_1]$ and $[Lb, N_2]$, respectively, then $[Lb, N_1] \cup [Lb, N_2]$ represents the larger of the two, i.e. $max(N_1, N_2) = N_2$. Thus, we can support negative numbers.

Minimum Costs Now, say Ub represents the upper bound for all positive numbers supported by the system architecture. We can represent the set of all numbers between N and Ub, i.e. the interval $[N, Ub]$, as the number N. Observe that a number smaller than N is represented by an interval that contains the interval $[N, Ub]$. As before, if we take a union of two or more such representations, the result is the *largest interval*. Using Figure 17(c) as an example, with N_1 and N_2 represented by $[N_1, Ub]$ and $[N_2, Ub]$, respectively, then $[N_1, Ub] \cup [N_2, Ub]$ represents the smaller of the two, i.e., $min(N_1, N_2) = N_2$.

As another example of these semantics, consider the first derivation in Figure 2 - $\texttt{spaths}(\texttt{a}, \texttt{c}, 2)$ was derived because the previous value for $(\texttt{a}, \texttt{c})$ was 3. In the interval semantics, $\texttt{spaths}(\texttt{a}, \texttt{c}, 2)$ would be represented as $[2, Ub]$, and 3 as $[3, Ub]$, thus we have $([2, Ub] \cup [3, Ub])$ is $\texttt{min}(2, 3) = 2$.

B. Normal Programs

$DeAL$ programs that only use monotonic arithmetic and monotonic boolean (comparison) functions on values produced by monotonic aggregates will be called *normal*[14]. All practical algorithms we have considered only require the use of *normal* $DeAL$ programs. Two classes of *normal* programs exist.

Class 1 This class of *normal* programs uses monotonic aggregates to compute the max (min) values for use outside the recursion (e.g. to return the facts with the final max (min) values). Programs in this class include Examples 1, 2 which use a stratified `max` (`min`) aggregate at the end of the fixpoint iteration to select the max (min) value produced by the recursive rules. Examining the intermediate results of Class 1 programs: at each step of the fixpoint iteration, we have (i) the max (min) value (ii) values less than the max value (greater than the min value). However, we do not need the values in (ii), as long as the values in the head are computed from those in the body via an *arithmetic function that is monotonic*.

Class 2 Values produced by monotonic aggregates in Class 2 *normal* programs are not passed to rule heads, but are tested against conditions in the rule body. Here too, as long as the functions applied to the values are monotonic, the rules are satisfied if and only if they are satisfied for the max (min) values. Example 7 is a Class 2 normal program.

Normal Program Evaluation Recall the algorithm for *MASN* in Figure 6. Let us call L the set produced by $T_E(M)$ or $T_R(\delta S)$, F the set produced from applying $getLast()$ to L. For *Semi-naive* δS will be L, whereas for *MASN*, δS will be F. Let $W = L - F$. $W = \varnothing$ when, for facts from monotonic aggregate predicates, each group with facts derived during the iteration has only one fact. For an iteration, if $W = \varnothing$, *MASN* evaluates the program the same as *Semi-naive*. Otherwise, W contains facts that will not lead to final answers for Class 1 *normal* programs and for Class 2 *normal* programs, any condition satisfied (in a rule body) by a fact in W will also be satisfied by the fact of the same group in F. Thus, *MASN* does not need to evaluate W. In the next iteration, *Semi-naive* will derive all of the same facts as *MASN*, but also derive facts evaluating W. We have already established that these facts, because they were derived from W, will not lead to final answers or to satisfying rule bodies, and thus, *MASN* does not need to derive facts with these either.

Theorem 11.1: A *normal* program with monotonic aggregates evaluated using *MASN* will produce the same result as that program evaluated using *Semi-naive*.

Recall the algorithm for *MASN* in Figure 8. Assume we are evaluating a *normal* program with *Semi-naive*. Let us call K_{sn}

[14]The compiler can easily check if a program is normal when the program contains only arithmetic and simple functions (e.g. addition, multiplication).

the set of facts used in derivations ($\cup$ of all δS) by *Semi-naive*. Now assume we are evaluating the same *normal* program with *EMSN*. Let us call K_{emsn} the set of all facts used in derivations by *EMSN*, which means K_{emsn} contains facts that were retrieved from the aggregate's relation, meaning they had the current value for the group at the time the fact was used in derivation. Now, let $C = K_{sn} - K_{emsn}$. If $C = \varnothing$, *EMSN* evaluates the program the same as *Semi-naive*. Otherwise, C contains facts that were not used by *EMSN* because at the time of derivation, the values in the aggregate's relation for those facts' groups were greater (`mmax`, `mcount`, `msum`) or lesser (`mmin`) than the aggregate value in the fact. We know $K_{sn} \cap K_{emsn} = K_{emsn}$ and $K_{emsn} \subset K_{sn}$ because *EMSN* will ignore facts to use in derivations that *Semi-naive* will use in derivations, but *EMSN* will not use different facts than *Semi-naive*. Stated another way, *Semi-naive* will attempt derivations with all facts *EMSN* attempts derivations with. Therefore, for Class 1 *normal* programs, each fact in C was not going to lead to final answers. For Class 2 *normal* programs, any condition satisfied (in a rule body) by a fact in C would have also been satisfied by the value that was instead used. *EMSN* does not need to evaluate C. We have:

Theorem 11.2: A *normal* program with monotonic aggregates evaluated using *EMSN* will produce the same result as that program evaluated using *Semi-naive*.

XII. Related Work

We have discussed the contributions of many works including [2], [9], [23] in previous sections. Focusing on extrema aggregates, [25] proposes rewriting programs by pushing the aggregate into the recursion and using *Greedy Fixpoint* evaluation to select the next min/max value to execute. [26] proposes rule rewriting using *aggregate selections* to identify *irrelevant* facts that are to be discarded by an extended version of *Semi-naive* evaluation. Nondeterministic constructs and stable models are used to define monotone aggregates in [27]. The Bloom^L language [16] for distributed programming uses logical monotonicity via built-in and user-defined lattice types to support eventual consistency.

XIII. Conclusion

With the renaissance of Datalog, the monotonicity property has been placed at the center of its ability to provide a declarative treatment of distributed computation [30]. In this paper, we have shown how this property can be extended to the aggregate involved in recursive computations while preserving the syntax, semantics, and optimization techniques of traditional Datalog. The significance of this result follows from the fact that this problem had remained long unsolved, and that many new applications can be expressed with the proposed extensions that make them amenable to parallel execution on multiprocessor and distributed systems. Future lines of work include supporting KDD algorithms and parallel/distributed implementations of techniques from this paper.

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